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Gravity Recovery and Climate Experiment

# Level-2 Gravity Field Product User Handbook

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# I BACKGROUND

## I. 1 PURPOSE OF THE HANDBOOK

The GRACE Level-2 Gravity Field Product User Handbook provides a description of the Earth gravity field estimates provided by the Level-2 processing. In particular, the relationship of the abstract concept of the geopotential (i.e. Earth's gravity field and its variations), its instantaneous and mean values, and the specific set of GRACE gravity estimates (i.e. GRACE gravity field products) is elaborated.

## I. 2 GRACE MISSION

The primary objective of the GRACE mission is to obtain accurate estimates of the mean and time-variable components of the Earth's gravity field variations, for a period of five years. This objective is achieved by making continuous measurements of the change in distance between twin spacecraft, co-orbiting in ≈ 500 km altitude, near circular, polar orbit, spaced ≈ 220 km apart, using a microwave ranging system. In addition to this range change, the non-gravitational forces are measured on each satellite using a high-accuracy electrostatic, room-temperature accelerometer. The satellite orientation and position (& timing) are precisely measured using twin star cameras and a GPS receiver, respectively.

Spatial and temporal variations in the Earth's gravity field affect the orbits (or trajectories) of the twin spacecraft differently. These differences are manifested as changes in the distance between the spacecraft, as they orbit the Earth. This change in distance is reflected in the time-of-flight of microwave signals transmitted and received nearly simultaneously between the two spacecraft. The change in this time of flight is continuously measured by tracking the phase of the microwave carrier signals. The so-called dual-one-way range change measurements can be reconstructed from these phase measurements. This range change (or its numerically derived derivatives), along with other mission and ancillary data, is subsequently analyzed to extract the parameters of an Earth gravity field model.

## I. 3 THE GEOPOTENTIAL

The word "geopotential" in this document will refer to the exterior potential of the Earth system, which includes its entire solid and fluid (including oceans and atmosphere) components. Following conventional methods (Heiskanen & Moritz 1966), at a field point P, exterior to the Earth system, the potential of gravitational attraction between a unit mass and the Earth system may be represented using an infinite spherical harmonic series. The field point P is specified by its geocentric radius r, geographic latitude □, and longitude □. If □ represents the gravitational constant of the Earth, and $a_e$ represents its mean equatorial radius (or a scale distance), then the Earth's exterior potential (or geopotential, as used in this document) can be represented as

$$V(r,\phi,\lambda,t) = \frac{GM}{r} + \frac{GM}{r} \sum_{l=2}^{N_{\max}} \left(\frac{a_e}{r}\right)^l \sum_{m=0}^{l} \overline{P}_{lm}(\sin\phi) \left\{ \overline{C}_{lm}(t)\cos m\lambda + \overline{S}_{lm}(t)\sin m\lambda \right\} \qquad (1)$$

In this expression, $\overline{P}_{lm}(\sin\phi)$ are the (fully-normalized) Associated Legendre Polynomials of degree $l$ and order $m$; and $\overline{C}_{lm}$ and $\overline{S}_{lm}$ are the (fully-normalized) spherical harmonic coefficients of the geopotential.

The geopotential at a fixed location is variable in time due to mass movement and exchange between the Earth system components. This is reflected by introducing the independent variable time (t) on the left; and is implemented or realized by treating the spherical harmonic coefficients of the geopotential as time dependent. The continuum of variations of the geopotential is represented by theoretically continuous variation of the geopotential coefficients.

Though the spherical harmonic expansion of the geopotential requires an infinite series of harmonics, practicality dictates that the summation on the right be limited to a maximum degree $N_{max}$.

In satellite geodetic convention, the origin of the reference frame is chosen to be coincident with the center of mass of the entire Earth system, including its solid component and fluid envelopes. In this convention, the potential has no terms of degree $l=1$ on the right hand side of Eq. 1.

## I. 4 Normalization Conventions

If $\phi$ denotes the geographical latitude of a field point (0° at equator, 90° at the North pole, and –90° at the South pole), and if $u = \sin\phi$, then the un-normalized Legendre Polynomial of degree $l$ is defined by

$$P_l(u) = \frac{1}{2^l \cdot l!} \cdot \frac{d^l}{du^l}\left(u^2 - 1\right)^l$$

The definition of the un-normalized Associated Legendre Polynomial is then

$$P_{lm}(u) = \left(1 - u^2\right)^{\frac{m}{2}} \frac{d^m}{du^m} P_l(u)$$

If the normalization factor is defined such that

$$N_{lm}^2 = \frac{(2 - \delta_{0m})(2l+1)(l-m)!}{(l+m)!}$$

and the Associated Legendre Polynomials are normalized by

$$\overline{P}_{lm} = N_{lm} P_{lm}$$

then, over a unit sphere S

$$\int_S \left[ \overline{P}_{lm}(\sin\phi) \begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} \right]^2 dS = 4\pi \qquad (2)$$

In this convention, the relationship of the spherical harmonic coefficients to the mass distribution becomes

$$\begin{Bmatrix} \overline{C}_{lm} \\ \overline{S}_{lm} \end{Bmatrix} = \frac{1}{(2l+1)M_e} \iiint_{Global} \left( \frac{r'}{a_e} \right)^l \overline{P}_{lm}(\sin\phi') \begin{Bmatrix} \cos m\lambda' \\ \sin m\lambda' \end{Bmatrix} dM \qquad (3)$$

where $r'$, $\phi'$ and $\lambda'$ are the coordinates of the mass element dM in the integrand. The integration is carried out over the entire mass envelope of the Earth system, including its solid and fluid components.

This convention is consistent with the definition of fully-normalized harmonics in NRC (1997), and textbooks such as Heiskanen and Moritz (1966), Torge (1980); as well as in earlier gravity field models such as EGM96.

## I. 5 Component Variations

At several places in this document, the total geopotential, as quantified by a set of total, time-dependent spherical harmonic coefficients, is separated into its component variations. Each component might represent one or more parts of the total Earth system (e.g. atmospheric or oceanic variations), or might represent a specific geophysical phenomenon (e.g. solid Earth tides).

Such separate components are used in a linearly additive manner in all of GRACE data analysis. In particular, if a component has for its domain only a part of the Earth system, then the coefficients for that phenomenon represent the contribution to the *exterior* geopotential from an integration carried over a limited spatial domain in Eq. 3.

# II GEOPOTENTIAL ESTIMATES

## II. 1 INTRODUCTION

An instantaneous measurement of biased-range, or its derivatives, from the GRACE instruments suite, is directly related to the instantaneous position and velocity (or trajectory) of the GRACE spacecraft. The GRACE spacecraft trajectories contain the influence of the total exterior geopotential (and other forces) on each GRACE satellite. In the GRACE data analysis, the mathematical model for this dependence is the dynamical equations of motion for each satellite,

$$\ddot{\vec{r}}_A = \vec{f}_A \quad ; \quad \ddot{\vec{r}}_B = \vec{f}_B \qquad (4)$$

where the subscripts A and B denote the two GRACE satellites, $\vec{r}$ is the position vector in an Earth-centered inertial frame; and the right hand side is the sum of gravitational and non-gravitational accelerations acting on each satellite.

In principle, therefore, a collection of biased range measurements, over a suitable time span, with suitable or sufficient geographical coverage, and properly corrected for non-gravitational effects, is implicitly representative of the exterior gravity field of the Earth and its variations within that time span.

In GRACE science processing, this implicit relationship is parametrized in a specific way, and estimates of the geopotential parameters are determined from this selected data span. These estimates represent both improvements to the existing models for the Earth's gravitational variations as well as new information on previously un-modeled gravity variations due to various geophysical phenomena. This concept is elaborated in the following sections.

## II. 2 BACKGROUND GRAVITY MODELS

In each processing of the GRACE science data, estimates are made of updates to an *a priori* best-known geopotential model. This *a priori* gravity model is a part of the so-called Background Model. This approach is used for several reasons. The iterative, linearized least squares approach to the solution of an essentially non-linear problem is better behaved (in the sense of convergence) if the linearized updates are small, which is assured by using comprehensive background gravity models. Furthermore, due to orbital track coverage limitations, rapid variability in the gravity field cannot be well determined from GRACE data, though, if neglected, it has the potential to corrupt the GRACE estimate through aliasing. Finally, some geophysical models of variability can be better determined from other techniques besides GRACE, in which case it is beneficial to remove such signals from the background using the best available external knowledge.

The Background Model consists of mathematical models and the associated parameter values, which are used along with numerical techniques to predict a best-known value for the observable (in this case, the inter-satellite range or its derivatives). The Background Model encompasses both satellite dynamics and measurements. This model may be expected to change with the evolution of processing methods – and is described in detail in the respective *Level-2 Processing Standards Documents.*

Since the geopotential is parametrized as a set of spherical harmonic coefficients, let G(t) represent the value of any coefficient set at epoch *t*. The Background Gravity Model value may then be written as

$$G^*(t) = \overline{G}^* + \dot{G}(t - t_0) + \Delta G^{st}(t) + \Delta G^{ot}(t) + \Delta G^{pt}(t) + \Delta G^{a+o}(t) \qquad (5)$$

The first term $\overline{G}^*$ represents the *a priori* best knowledge of the static (or long-term mean) geopotential. It represents most of the non-spherical gravitational forces acting on the satellite, and in the past has been represented by models such as EGM96. It is possible for this to be one of the previously derived GRACE gravity field products, though it is not required to be so.

The remaining terms in Eq. (5) represent the *a priori* best knowledge of the variations in the gravity field of the Earth. The second term represents the secular variations of the harmonics; the third, fourth and fifth terms represent the solid, ocean & pole tides, respectively. The last term a combination of atmospheric and oceanic non-tidal variability.

The background gravity model used for data processing differs from the true gravity field of the Earth in two ways – by errors of omission of either certain geophysical phenomena or spatial components; and by errors of commission by having incorrect models or parameter values for certain other phenomena.

### *II.2.1 Mixed Resolution of Background Models*

It is noted that each of the component background models has its own inherent resolution in time and space. Thus, the secular rates are available and significant for only the lowest degree harmonics. The body-tides and ocean-tides are semi-analytic functions – where the variations occur at discrete & well-known frequencies, and are calculable at any arbitrary epoch, but have empirically determined amplitudes at variable spatial resolutions. The pole-tides have the resolution of the polar-motion time series and are evaluated at intermediate epochs by suitable interpolation. The non-tidal atmosphere and oceans have different resolution in time and space, and must be similarly interpolated at intermediate epochs. An example of the component variations over three arbitrarily selected days is shown in Figure 1.

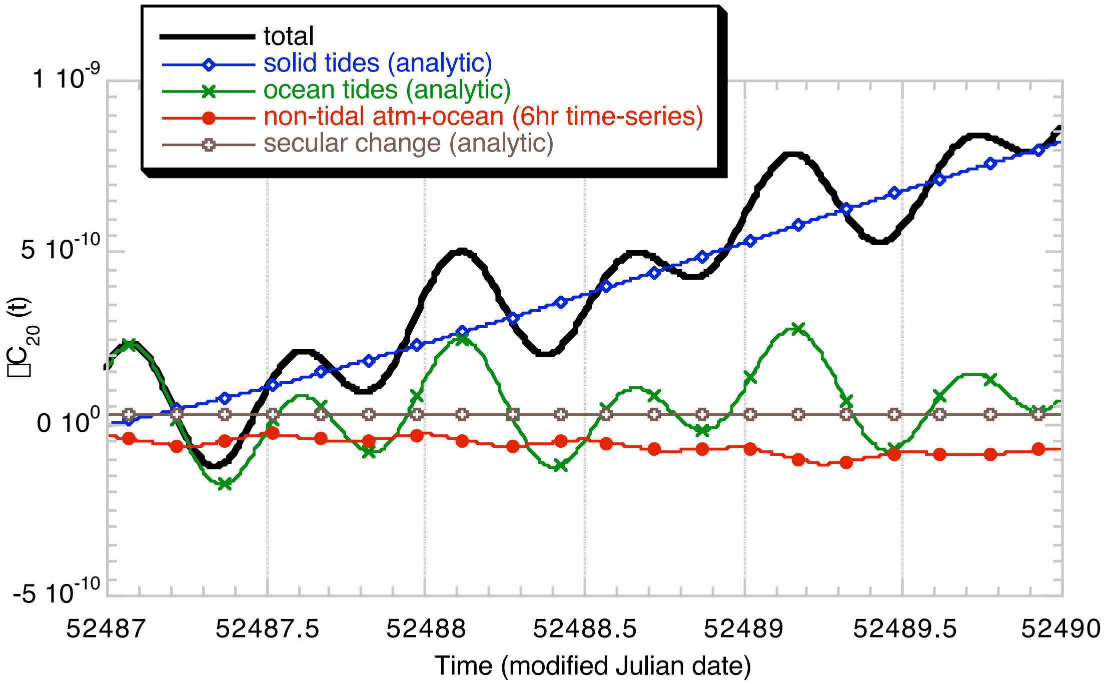


**Figure 1 Components of the background model contributions to the C20 harmonic.**

## II. 3 Parametrization of Geopotential Estimate

The collection of background models is used in GRACE data processing to make a prediction of the observable range change (or its derivatives). The difference between the observed and predicted values of the measurements is the residual, which exists because of the aforementioned errors of omission & commission, in addition to the measurement errors and model deficiencies. The background gravity model errors may be expected to have continuum spatio-temporal variability. An update to the background gravity model is to be computed such that the measurement residuals are minimized in the least-squares sense – and this update may be regarded as the new gravity information available from GRACE.

In GRACE science data processing, at the most elementary level, this update to the geopotential is parametrized, for a selected data span, as a set of constant corrections to the spherical harmonic coefficients of the geopotential, to a specified maximum degree and order. Functionally, this may be represented as

$$\Delta\hat{G}(T_s) = L\left\{Y_i - f\left(G^*(t_i)\right), i = 1,\ldots,m\right\} \qquad (6)$$

In this equation, the left hand side represents a set of geopotential parameters estimated from a specific data span $T_s$; L stands for the linearized least squares problem solved to obtain this estimate, using a collection of *m* observation residuals within that data span. For the specific coefficient $\overline{C}_{20}$, the situation is depicted pictorially in Figure 2.

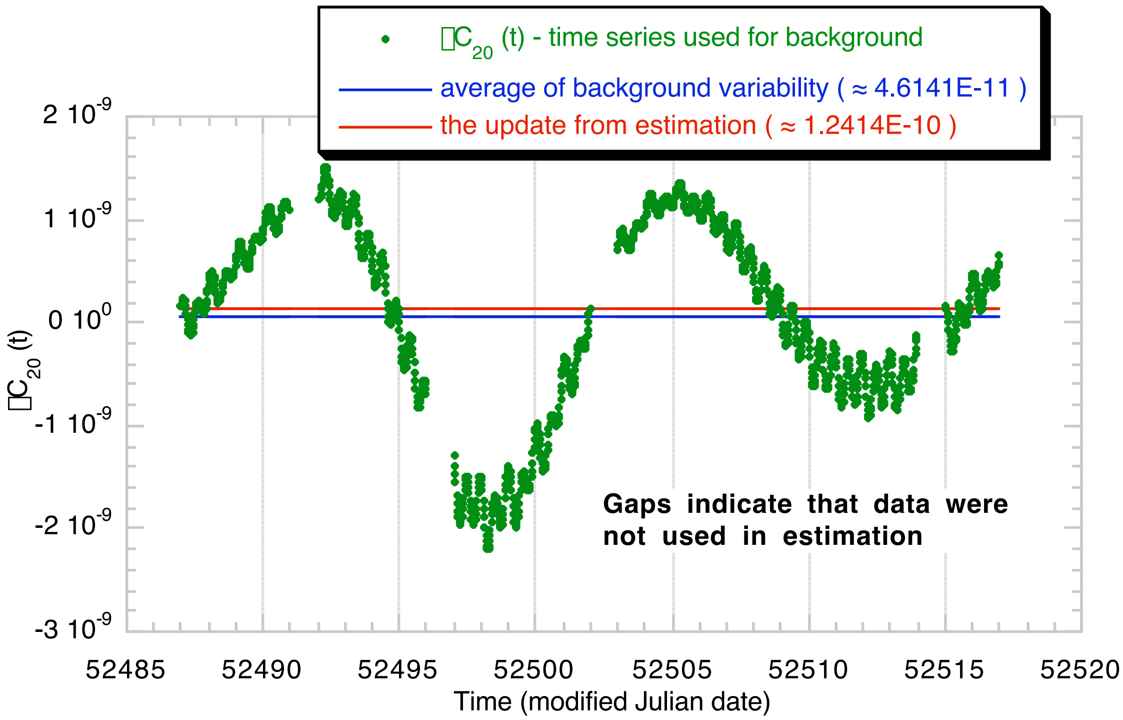


**Figure 2 Depiction of the background model and its update from a GRACE solution.**

In this way, the long term variability of the geopotential relative to the adopted background gravity model is traced by a sequence of piece-wise constant (or comparable) spherical harmonic coefficient sets over a specific duration – generally on the order of one month. Note in Figure 2 that not every data point within the span may have been used for the estimation of the constant update to the geopotential – even though the estimate is said to be valid over a data span bound by its end points.

As stated before, the sequence of such GRACE derived gravity estimates represent not only the corrections to the existing models for the Earth's gravitational variations, but also new information on previously un-modeled gravity variations due to various geophysical phenomena

### *II.3.1 The Geopotential Product*

For several applications, it is useful to regard the estimate $\Delta\hat{G}(T_s)$ as an update or correction to the background static geopotential model. This is also consistent with historical practice in geodesy. For this reason, both $\Delta\hat{G}(T_s)$ as well as the quantity

$$\tilde{G}(T_s) = \overline{G}^* + \Delta\hat{G}(T_s) \qquad (7)$$

are to be regarded as GRACE data products – and this distinction is clear in the product nomenclature. Either of these products represents corrections to errors of commission or omissions from the background gravity models, and represents equally the new gravity information available from GRACE for scientific analysis.

### *II.3.2 Epochs Associated With the Geopotential Product*

In general, there is no single epoch that can be perfectly associated with the GRACE Level-2 gravity field product. The start and the stop times for the data span $T_s$ are provided along with the coefficients values as a part of the Level-2 products. The estimates for any one span may be treated as corrections to the average of the background gravity model for that span. If it is necessary to associate a gravity field estimate with a single epoch, the mid-point of the data span used in the solution may be adopted.

## II. 4 THE AVERAGE GEOPOTENTIAL

As is evident from previous discussions, for the data span $T_s$, the total geopotential is represented as a heterogeneous mix between the background gravity model and the geopotential estimate update. In particular, orbital dynamics and the estimation process are necessarily non-linear, so that the changes in the background gravity models do not necessarily connect linearly to changes in the estimated piece-wise constant updates to the geopotential.

Nevertheless, to some approximation and particularly for small variations, it is possible to relate the averages of background gravity models and the estimate updates. Define the time-average of the geopotential coefficients by

$$\langle G \rangle^{T_s} \equiv \frac{1}{T_s} \int_{T_s} G(\tau)\, d\tau \qquad (8)$$

Recalling Eq. 5, over the specific data span $T_s$, let $\left\langle \delta G^{bg} \right\rangle_o^{T_s}$ represent the average of the errors of omission in the background gravity model; let $\left\langle \delta G^{bg} \right\rangle_c^{T_s}$ represent the average of the errors of commission in the background gravity model; and let $\delta \overline{G}^*$ represent the errors in the background mean or static gravity field.

Then the best estimate of the geopotential update from GRACE data processing over the time-span $T_s$ may be written as

$$\delta \hat{G}(T_s) \approx \left\langle \delta G^{bg} \right\rangle_o^{T_s} + \left\langle \delta G^{bg} \right\rangle_c^{T_s} + \delta \overline{G}^* \tag{9}$$

The expression does not use an equality operator as the two sides differ by the estimation errors in GRACE data processing.

### *II.4.1 Interpreting Product Variability*

Let the best estimates for two different data spans $T_A$ and $T_B$ be written as

$$\begin{aligned} \delta \hat{G}(T_A) &\approx \left\langle \delta G^{bg} \right\rangle_o^{T_A} + \left\langle \delta G^{bg} \right\rangle_c^{T_A} + \delta \overline{G}^* \\ \delta \hat{G}(T_B) &\approx \left\langle \delta G^{bg} \right\rangle_o^{T_B} + \left\langle \delta G^{bg} \right\rangle_c^{T_B} + \delta \overline{G}^* \end{aligned} \tag{10}$$

This pair of equations provides a possible basis for interpretation of a sequence of GRACE products as time variable signals in the Earth gravity field, to within the combined errors of estimation from the two data spans.

If one assumes that the background gravity models are perfect except for errors of omissions, (i.e. errors of commission are zero), the difference between GRACE products from two different data spans is interpretable as the change in the average of the gravity signals due to the omitted variability effects. In that case,

$$\delta \hat{G}(T_A) - \delta \hat{G}(T_B) = \left\langle \delta G^{bg} \right\rangle_o^{T_A} - \left\langle \delta G^{bg} \right\rangle_o^{T_B} \tag{11a}$$

If some or all of the background gravity model components are deemed imperfect (i.e. errors of both omission & commission are non-zero), then the estimate differences additionally reflect the change in the average of the errors of commission between the two data spans. In this case,

$$\begin{aligned} \delta \hat{G}(T_A) - \delta \hat{G}(T_B) \approx{} & \left\langle \delta G^{bg} \right\rangle_c^{T_A} - \left\langle \delta G^{bg} \right\rangle_c^{T_B} \\ & + \left\langle \delta G^{bg} \right\rangle_o^{T_B} - \left\langle \delta G^{bg} \right\rangle_o^{T_A} \end{aligned} \tag{11b}$$

Note that, in general, no frequency domain filtering is done on the input background models. Thus the GRACE estimate updates reflect the averages of the omission and commission error signals with the complete frequency bandwidth of the phenomena.

This discussion also provides a guide if the user wishes to mimic approximately the effects of replacing a component of the background model on the variability evident in the GRACE products. Nominally, the estimate for any one span is an update relative to the average of the background gravity model. The replacement background gravity model may not be expected to have the same average. In this case, the right-hand side must have an additional term that takes into account the difference of the averages of the original and replacement background model over the two data spans.

The GRACE product taxonomy has been developed to accommodate the definitions and combinations described in this section. The preceding discussion is applicable to both forms of the product described in II.3.1.

# III GRACE LEVEL-2 PRODUCTS

## III. 1 PRODUCT INDENTIFIER

A GRACE Level-2 gravity field product is a set of spherical harmonic coefficients of the exterior geopotential. A product name is specified as

*PID-2_dddd_YYYYDOY-YYYYDOY_sssss_mmmm_rrrr*

Where

*PID* is 3-character product identification mnemonic
*-2* denotes a GRACE Level-2 product
*dddd* is the (leading-zero-padded) number of calendar days from which data was used in creating the product.
*YYYYDOY-YYYYDOY* specifies the date range (in year and day-of-year format) of the data used in creating this product
*sssss* is an institution specific string
*mmmm* is a 4-character free string (e.g. used for specifying multi-satellite solutions)
*rrrr* is a 4-digit (leading-zero-padded) release number (0000, 0001, … )

The Product Identifier mnemonic (PID) is made up of one of the following values for each of its 3 characters:

*1st Character*

= G: Geopotential coefficients

*2nd Character*

= S: Estimate made from only GRACE data
= C: Combination estimate from GRACE and terrestrial gravity information
= E: Any background model specified as a time-series
= A: Average of any background model over a time period

*3rd Character*

= M: Estimate of the Static field.
This is the quantity $\tilde{G}(T_s)$ from Eq. 7 in Section II.3.1.
(The data files for this product also contain records with the epochs & rates used to model secular changes in the background gravity model)
= U: Geopotential estimate relative to the background gravity model
This is the quantity $\Delta\hat{G}(T_s)$ from Section II.3.1.

= T: Total background gravity model except for background static model
This is based on the quantity $\left(G^{*}(t) - \overline{G}^{*}\right)$ from Eq. 5 in Section II. 2

= A: non-tidal atmosphere (see *AOD1B Description Doc*)

= B: non-tidal Oceans (see *AOD1B Description Doc*)

= C: combination of non-tidal atmosphere and ocean -
for details of combination see *AOD1B Description Doc*

Not all possible combinations of characters make sense, or are provided as products. As of this release, the list of defined products, and the associated format names, is summarized in Table 1.

| PID | Format | Remarks |
|---|---|---|
| GSM | GRCOF2 | |
| GCM | GRCOF2 | |
| | | |
| GEA | AOD1B | *This is same as product AOD1B* |
| GEB | AOD1B | *This is same as product AOD1B* |
| GEC | AOD1B | *This is same as product AOD1B* |
| | | |
| GAC | GRCOF2 | |
| | | |
| | | |

**Table 1 Product Identifiers and associated formats**

## III. 2 PRODUCT CONTENTS

In general, all products that share a format have the same definition for the content. Some details are provided in this section.

### *III.2.1 Defining Constants*

Constants such as □ or $a_e$, or other parameters that are necessary for reconstructing the geopotential or its derivatives are always provided in the product file header records.

### *III.2.2 Product Span and Epoch*

The product name includes a range of epochs. The start date is the date of the first observation or data point used in the processing, and the end date is the date of the last data point used. The dates are tracked to at least 1-day discretization, but it is possible for the first or the last observation to fall at any time within that day. Finally, it is not assured that all data within the spans in the product identifier were used in the processing.

N.B. Specifically for the GSU or GCU products: Within a fixed data span, identified by the strings in the product name, it is possible that certain selected geopotential parameters may be solved as piece-wise constants over sub-intervals of the overall data span. In this circumstance, within one product, some coefficients may appear multiple times in different records – with a unique span of sub-epochs distinguishing such parameters.

### *III.2.3 Calibration Coefficients for Uncertainties*

For the products GSx or GCx, a scale factor is provided such that calibrated standard deviation = scale_factor * formal standard deviation.

### *III.2.4 Companion Documents*

Complete specification of the background models and conventions is given in the respective *Level-2 Processing Standards Documents*. Each data release is associated with a corresponding update of this document.

The formats mentioned in Table 1 are contained in the *Level-2 Formats Document*.

# REFERENCES

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