GRACE Education Curriculum Satellites	
Teachers	Grades 9-12
Physics, Pre-Calculus & Calculus	

What If Kepler Had a Graphing Calculator?

Background Information: At the time of Johannes Kepler (1571-1630), most scientists believed that the planets moved in circular orbits. But circular orbits could not satisfactorily explain the observed movements of the planets. So Kepler formulated other hypotheses to explain planetary motion. The one that best fit the observations was: All the planets have elliptic orbits. Kepler was convinced that geometry and mathematics could be used to explain the number, distance, and motion of the planets. By doing careful analysis of data, Kepler discovered three laws that still describe the behavior of every planet and satellite. It took Johannes Kepler ten years of painstaking effort to discover the relationship between the period around the sun and the radius of the planet.

Objectives: The student will:

- Investigate Kepler's third law of planetary motion.
- Be able to recognize from a selection of graphs the best description of data and relationship of Kepler's third law.
- Successfully input data into the graphing calculator to produce scatter plots with the lines of regression.

Standards: Science: unifying concepts & processes; physical science; earth and space science

Vocabulary: planetary motion elliptical orbit lines of regression

Materials: Graphing calculator or computer Data Chart
Questionnaire Labels (cards)

Procedure:

1. Provide background information to the students about Kepler and his laws of planetary motion.
2. Review the instructions for using the graphing calculator. Have the students become familiar with the functions of the graphing calculator. Distribute a copy of the instructions for inputting data in the TI-83/TI-83 Plus (you may or may not wish to hand out.) The instructions can be found in the TI-83 Guide Book on page 12-10. Instructions for scatter plots are found on page 12-31. The instruction for producing a line of regression is found on page 12-22.
3. Distribute the data chart to the students. Have students calculate t-squared (period squared), t-cubed (period cubed), r-squared (radius squared), and r-cubed (radius cubed). Note: if time is limited you may want to either supply a completed data chart or assign the data previous to the lesson.
4. Once all students have a completed data chart, you may wish to discuss significant digits. How many digits are significant for this data? (Note: The graphing calculator automatically rounds when data is entered into a list to a significant digit.)
5. Have the students work in groups or independently. Your goal is to get one to two complete sets of the nine possible graphs. It is best to have the 9 separate graphers displayed at one point in the room so all students can compare the 9 graphs side-by-side to determine the best representation of the data. Each grapher will need a label (card) denoting what relationship is being graphed. (i.e.: t vs. r, t vs. r-squared, etc.)
6. Distribute the questionnaire to each student. As the students compare the 9 graphs, have them complete the questions.
7. The goal is for the students to see that t-squared vs. r-cubed give the most linear representation of the data. Thus discovering Kepler's third law.
8. Explain to the students that they will make a straight line graph where T is the period of the planet or time it takes for the planet to orbit the sun. R is the radius of the planet. A linear (straight-line) relationship shows that the two variables are directly proportional to each other. Plot T vs. R

- Is the graph linear?

9. If the answer to the question is no, try plotting T vs. R^2, T vs. R^3 and R vs. T, R vs. T^2, R vs. T^3, etc.

10. If the relationship between T and R cannot be discovered by modifying only one variable at a time, try modifying the powers of both variables at the same time.
    - What relationship finally gave you a linear relationship?

Try this to enter the data.

$x \rightarrow L_1$

$y \rightarrow L_2$

$x^2 \rightarrow L_3$ Place cursor on $L_3 \rightarrow L_1$ (x^2)

$x^3 \rightarrow L_4$ Place cursor on $L_4 \rightarrow L_1$ (math): 3

$y^2 \rightarrow L_5$ Place cursor on $L_5 \rightarrow L_2$ (x^2)

$y^3 \rightarrow L_6$ Place cursor on $L_6 \rightarrow L_2$ (math): 3

Stat Plot

(2nd)(y=)

1: Plot 1: On : Type (graph)

To Graph:	x list	y list
(x,y)	L_1	L_2
(x^2, y)	L_3	L_2
(x^3, y)	L_4	L_2
(x, y^2)	L_1	L_5
(x, y^3)	L_1	L_6

etc.

Zoom 9 to graph after each pair zoom: 9 (zoom stat)

If you write an equation, you must have the data you want in L_1 and L_2 for (x,y)

The correct coordinates are period squared vs. radius cubed.

Extensions:

- Research Johannes Kepler and his studies of planetary motion.
- Develop a timeline of planetary motion discoveries.
- Compare and contrast Kepler's laws of planetary motion with other scientists of his time.

References / Resources:

Kepler, Johannes. Epitome of Copernican Astronomy & Harmonies of the World, Prometheus Books, Amherst, New York.

http://encarta.msn.com/find/Concise.asp?ti=02F84000

http://sirius.phy.hr/~dpaar/fizicari/xkepler.html

DATA CHART

Planet Name	(years compared to Earth Years) Period t	Period Squared t^2	Period Cubed t^3	Average Radius r	Radius Squared r^2	Radius Cubed r^3
Mercury	.241			.39		
Venus	.615			.72		
Earth	1	1	1	1	1	1
Mars	1.88			1.52		
Jupiter	11.8			5.20		
Saturn	29.5			9.54		
Uranus	84.0			19.18		
Neptune	165			30.06		
Pluto	248			39.44		

DATA CHART

Planet Name	(years compared to Earth Years) Period t	Period Squared t^2	Period Cubed t^3	Average Radius r	Radius Squared r^2	Radius Cubed r^3
Mercury	.241	.05801	.01400	.39	.1521	.05932
Venus	.615	.37823	.23261	.72	.5184	.37325
Earth	1	1	1	1	1	1
Mars	1.88	3.5344	6.6447	1.52	2.3104	3.5118
Jupiter	11.8	139.24	1643.0	5.20	27.04	140.60
Saturn	29.5	870.25	25672	9.54	91.012	868.25
Uranus	84.0	7056	59270	19.18	367.87	7055.8
Neptune	165	27225	44921	30.06	903.60	27162
Pluto	248	61505	15253	39.44	1555.5	61349

Questionnaire

Name: _____

Answer the following for each graph

1. Is the graph linear?

1.	2.	3.
4.	5.	6.
7.	8.	9.

2. Which graph has the fewest points closest to the line of regression?

3. Which graph has the most points closest to the line of regression? (no points on the line)

4. Which graph has the most points on the line?

5. Which graph would appear to have the most linear relationship? (i.e.: t vs. r, t^2 vs. r, etc)

6. Why did we ask "appear" to have the most linear relationship?

Answer Sheet

1. Answer will depend of student graph
2. If the answer is no, try plotting T vs. R_, T vs. R_and R vs. T, R vs. T_, etc
3. If the relationship between T and R cannot be discovered by modifying only one variable at a time, try modifying the powers of both variables at the same time.