



GRACE Education Curriculum Satellites	
Teachers	Grades 9-12
Physics, Pre-Calculus, Calculus & Geometry	

Elliptically Speaking

Background Information: In early times, astronomers assumed that the orbits in which the planets moved were circular; yet the numerous catalogs of measurements compiled especially during the 16th century did not fit this theory. At the beginning of the 17th century, Johannes Kepler stated three laws of planetary motion that explained the observed data: the orbit of each planet is an ellipse with the sun at one focus; the speed of a planet varies in such a way that an imaginary line drawn from the planet to the sun sweeps out equal areas in equal amounts of time; and the ratio of the squares of the periods of revolution of any two planets is equal to the ratio of the cubes of their average distances from the sun. The path that planets take around the sun is an ellipse. An ellipse is an oval-shaped curve for which the distances from any point on the curve to two fixed points (the foci) in the interior have a constant sum. One way to draw an ellipse is to place a loop of string around two thumbtacks and pull the string taut with a pencil. Then slide the pencil along the string, keeping it taut.

In this work, Halley applied Newton's laws of motion to all available data on comets and then mathematically demonstrated that comets move in elliptic orbits around the sun. Manufactured satellites will have either an elliptical or circular orbit. A circle is actually an ellipse with zero eccentricity. For a circular orbit, the semi-major axis, the semi-minor axis, the radius of perigee, and the radius of apogee are all equal.

Objectives: To increase the students' understanding of the shapes that orbits can take.

Standards: Physical Science, Earth & Space Science

Vocabulary:

semi-major axis	Perigee	ellipse
semi-minor axis	apogee	circle

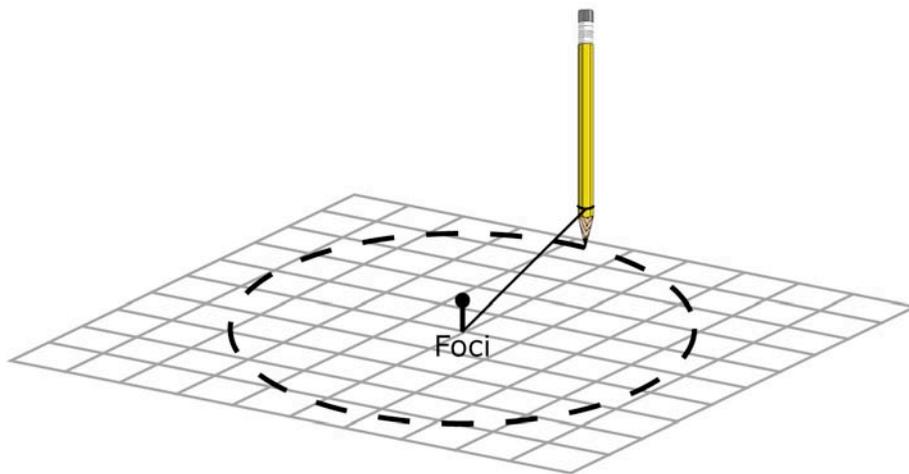
Materials:

2 thumbtacks or small pieces of masking tape	20 cm of string
piece of cardboard	graph paper
colored pencils	

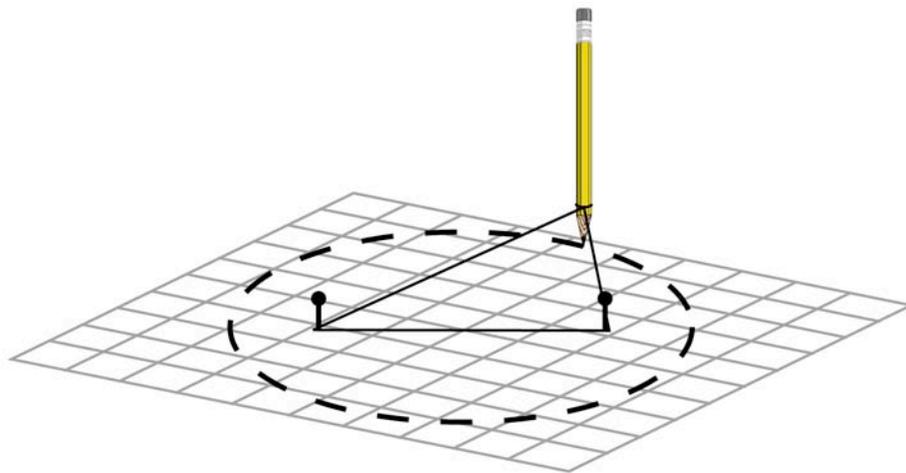
Directions to the Teacher:

Have the students complete the Elliptical Orbits handout. Once completed, conduct the following experiment:

1. Place a piece of graph paper on top of the cardboard. Push a thumbtack in the middle of the graph paper through the graph paper into the cardboard. Label this the foci. Make a loop with the string and place it over the thumbtack. Put the pencil in the loop and pull it taut. Draw a circle by keeping the pencil taut and drawing a 360° arc around the thumbtack. You may have to push down on the thumbtack to keep the string on it.



2. Now put two thumbtacks into the graph paper about 4 inches from the center and again loop the string around both. Using another colored pencil draw the ellipse just as you did the circle.



3. Locate the foci so that you can draw an even longer ellipse.
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4. A straight line is a special case of an ellipse. Determine the positions of the foci that would give you a straight line, and construct a straight line.

Directions to the Students: Analyze the following:

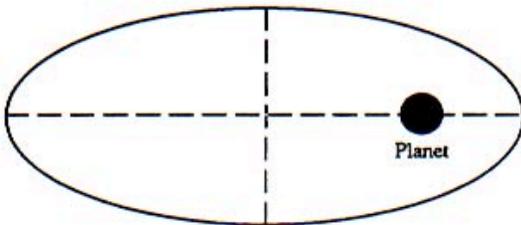
1. Which of your drawings is closest to the earth's orbit around the sun?
2. Which of your drawings is closest to the orbit of Halley's comet around the sun?
3. Which of your drawings is closest to the orbit of man-made satellites around the earth?
4. Complete the orbit simulation at: <http://www.usafa.af.mil/dfp/cockpit-phys/gp2ex4.htm>

Suggested References / Resources

Elliptical Orbits published by TSGC

Elliptical Orbits

Many satellites have orbits that look like ellipses, or ovals. Using the definitions given below, label the parts of an elliptical orbit on the diagram.



Definitions

semi-major axis (a): the distance between the center of the ellipse and the outer edge in the long direction

semi-minor axis (b): the distance between the center of the ellipse and the outer edge in the short direction

radius of perigee (r_p): the distance from the center of the planet to the point of closest approach for the satellite

radius of apogee (r_{ap}): the distance from the center of the planet to the point in the orbit farthest from the planet

eccentricity (e): the measure of how “squashed” an ellipse is; i.e. the distance between the center of the planet and the center of the ellipse

Fun Facts

- A satellite can either be natural like the Moon or artificial like the Hubble Space Telescope.
- A circle is actually an ellipse with zero eccentricity ($e=0$).
- For a circular orbit, the semi-major axis, the semi-minor axis, the radius of perigee, and the radius of apogee are all equal ($a=b=r_p=r_{ap}$).
- People who plan satellite orbits study **orbital mechanics**.
- If you are interested in the motions of satellites, planets, or exploratory spacecraft (like Voyager), you should plan on being an **aerospace engineering** major in college.

If you know an orbit has an eccentricity of 0.1 and a semi-major axis of 40,000 km, use the equations below to calculate these quantities:

1. $r_p =$ _____

2. $r_{ap} =$ _____

3. $e =$ _____

4. $a =$ _____

5. $b =$ _____

$$a = \frac{(r_{ap} + r_p)}{2}$$

$$e = \frac{(r_{ap} - r_p)}{(r_{ap} + r_p)}$$

$$r_p = a(1 - e)$$

$$r_{ap} = a(1 + e)$$

$$b = \sqrt{r_{ap} r_p}$$